Syllabus for Six-Month Pre-registration Course Work of Ph. D. Programmmes in Mathematics

Krishnaguru Adhyatmik Visvavidyalaya

Nasatra, Barpeta, Assam

COURSE STRUCTURE & EXAMINATION PATTERN OF THE Ph.D. PROGRAMME IN MATHEMATICS

OPTIONAL-I (Any one of the following)

1. Group A : Fundamentals of Game Theory
2. Group B : Linear Algebra and Real Analysis

OPTIONAL-II (Any one of the following)

1. Group A : Mathematical Theory of Evidence
2. Group B : Traffic Thoery and Graph Measures
3. Group C : Advanced Functional Analysis

ASSIGNMENT/TERM PAPER

(Under guidance of the prospective Supervisor concerned)

Course II

Group (A): Fundamentals of Game Theory

Marks: 80

Unit 1 : Basics of Game Theory

Historical background; Zero sum games; non-zero sum games; extensive form games; Cooperative games; Bargaining games; Cooperative versus non-cooperative games;

Unit 2 : Finite Two person Zero sum games

Saddle point; Minimax and maximin strategies; Solving 2 x n and m x 2 games; Dominance; Mixed strategy; Linear Programming Methods to solve a two person zero sum game.

Unit 3: Finite Two Person non-zero sum games

Basic Definitions; Nash equilibrium; Pure and mixed strategies in Nash equilibrium.

Unit 4 : Finite Extensive Form Games

The Extensive Form; The Strategic Form; Backward induction and subgame perfection; Perfect Bayesian equilibrium.

Unit 5 : Cooperative Game Theory Models

Cooperative Games with Transferable Utility; The Core; The Shapley value; The Nucleolus.

Textbook:

"Hans Peter, Game Theory - A Multi level Approach, Springer, 2008."

Reference book:

"R.P Gilles, The Cooperative Game Theory of Networks and Hierarchies, Springer 2008".

Course –II

Group (B): Linear Algebra and Real Analysis

Marks: 80

Unit 1 : Linear Algebra

Vector spaces: subspaces, sums and direct sums; finite dimensional vector spaces: bases and dimensions; Linear maps: null-spaces and range, invertibility; polynomials with real and complex coefficients; Eigenvalues and eigen vectors: triangularization and diagonalization of operators on finite dimensional vector spaces; Inner- product spaces: orthonormal bases, linear functional and adjoins; Operators on inner-product spaces: self-adjoint and normal operators, minimal polynomial, Jordan form; Traces and determinants of operators and matrices.

Unit 2 : Real Analysis

Completeness properties of real numbers, countable and uncountable sets, cardinality. Norms and metrics: Metric spaces, convergence of sequences, completeness, connectedness and sequential compactness; Continuity and uniform continuity; sequences and series of functions, uniform convergence, equicontinuity, Ascoli's theorem, Weierstrass approximation theorem, power series. Calculus of functions of several real variables: Partial and directional derivatives, differentiability, Chain Rule, Taylor's theorem, Maxima and Minima, Lagrange multipliers, Inverse function theorem, Implicit function theorem.

Textbooks:

1. *Seymour Lipschutz, Linear Algebra, Schaum's Series, Tata McGraw Hill Edition, 2004*
2. *Tom M. Apostal, Methematical Analysis, Narasa Publishing House, 1997.*

Reference books:

1. *K. Hoffman and R. Kunze, Linear Algebra, Prentice Hall of India, 1996*
2. *W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw Hill, 1976.*

Course –III

Group A: Mathematical Theory of Evidence

Marks:80

Unit-1: Basic Concepts:

Uncertainty-a taxonomy, Sources of Uncertainty, Nature of Uncertainty, Various Approaches of uncertainty Modeling, Basic Concepts of Evidence Theory, Basic Probability Assignment, Belief and Plausibility Measure and their Properties.

Unit-2: Combination of Evidences:

Dempster's rule of Combinations, Discount + Combine rule, Yager's Rule, Inagaki's rule, Zhang's rule Dubois and Prade rule, Mixing and Averaging rule, other combination rules.

Unit-3: Measure of Uncertainty and Generation of BPA:

Nonspecificity and Strife. Different methods to obtain BPA.

Unit-4: Applications:

Applications of Dempster-Shafer theory of evidence in real world problems.

Textbooks:

1. ***A Mathematical Theory of Evidence by Glenn Shafer, Princeton University Press.***
2. ***Measurement Uncertainty: An Approach via Mathematical Theory of Evidence by Simona Salicone, Springer.***

Reference Book:

"Uncertainty Modeling and Analysis in Engineering and the Sciences by B.M. Ayyub and G.J. Klir, Chapman & Hall/CRC."

Course : III

Group B : Traffic Theory and Graph Measures **Marks: 80**

Unit 1: Preliminaries

Basics of Graph Theory, Network approach to Nature, Four general classes: Technological networks, Social Networks , Information networks, and Biological networks, Random Networks, Real-world Networks, Examples.

Unit 2: Models of Complex Network

Generalized Random Graphs, Milgram Experiment ,Six degrees of separation, Small world Network, Old Tales of Tails, New Tales of Tails, Power Laws, Preferential Attachment Model.

Unit 3 : Measures and Metrices

Centrality Measures, Spectral Measures: Degree Distribution, Correlations, Shortest Path Lengths, Diameter, Closeness, Betweenness, Clustering Coefficient, Page Rank, Efficiency, Graph Spectra.

Textbooks:

1. ***M.V. Steen, Graph Theory and Complex Networks: An Introduction,2010.***
2. **R.Cohen, S.Havlin, Complex Networks: Structure, Robustness and Function, Cambridge University Press,2010.**

Reference Books:

1. ***E.Estrada, The Structure of Complex Networks: Theory and Applications, Oxford University Press,2011***
2. ***M.E,J.Newman, Networks: An Introduction, Oxford University, Press.2010.***
3. ***A-L Barabasi, Network Science, Cambridge Universtiy, Press 2016.***

Course : III

Group C : Advanced Functional Analysis Marks: 80

Unit I: Topological vector spaces, Separation properties, Finite-dimensional spaces, Metrization, Boundedness and continuity, Semi-norms and local convexity, Bilinear mappings, Convexity, Weak topologies, Compact convex sets.

Unit II: Basics of Banach algebra and C*algebra, Commutative Banach algebras and C*- algebras, Representation of C*-algebras.

Unit III: Spectral theory of linear operators in normed space, Spectral properties of bounded linear operators, Further properties of resolvent and spectrum, Compact linear operator on normed spaces with spectral properties, Spectral properties of bounded self-adjoint linear operators.

Unit-IV: Banach fixed point theorem, Applications to linear equation, Differential and Integral equations.

Textbooks:

1. ***Kreyszig E., Introductory Functional Analysis with Applications (John Wiley and Sons, New York, 1978).***
2. ***Rudin, W. Functional Analysis (McGraw Hill, 2000).***
3. ***Simmons, G. F. Introduction to Topology and Modern Analysis (Tata McGraw Hill Book Co. Ltd., 1963).***
4. ***Gerard J. Murphy, C* - Algebras and Operator Theory, Academic Press, Inc, 1990***

Reference books:

1. ***Ronald G. Douglas, Banach Algebra Techniques in Operator Theory, Second Edition, Springer-Verlag, New York, Inc, 1998.***
2. ***Limaye, B. V. Functional Analysis (Wiley Eastern Ltd., New Delhi, 1989).***

COURSE IV : Term Paper/DISSERTATION (Under guidance of the Supervisor concerned)

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